

## Chapter 3

# Optical Fibers – (Multimode) Basics

- How to conduct light ?
- Attenuation (衰减) ?
- Dispersion (色散) ?
- Bandwidth (带宽) ?

# History

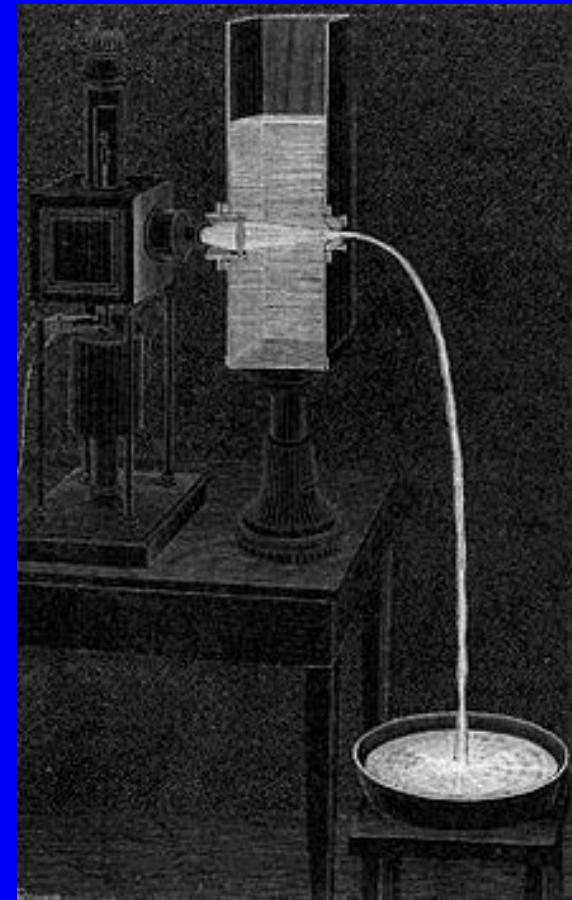
- Although glass fibers were made in the 1920s, their use became practical in the 1950s when the cladding layer was used.
- In 1966, 高鍬 suggested to reduce the loss of optical fiber for fiber-optic communications.
- In 1970, Corning Ltd. fabricated the step-index optical fiber.
- In 1972, the attenuation of optical fiber was improved from 20dB/km to 4 dB/km with the improvement of material and fabrication procedure.
- Further progress resulted by 1979, the loss of fiber was reduced to be only 0.2dB/km near the 1.55-um spectral region.



## 3.1 How optical fibers conduct light

Daniel Colladon first described this "light fountain" or "light pipe" in an 1842 article titled *On the reflections of a ray of light inside a parabolic liquid stream*. This particular illustration comes from a later article by Colladon, in 1884.

What's the principle?



## ◆ **Step-Index Fiber** (阶跃光纤)

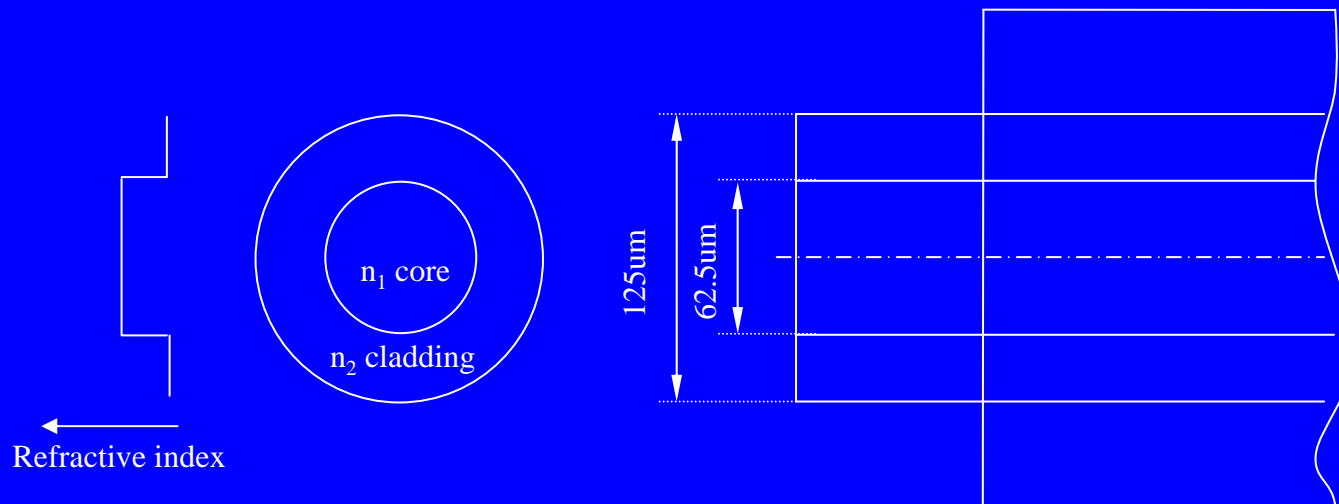
***Bare Fiber:*** Fiber Core + Cladding + Coating

*Core:* Doped silica  $\rightarrow n_1$

*Cladding:* Pure  $\text{SiO}_2 \rightarrow n_2$

*Coating:* Plastic or carbon

To achieve TIR:  $n_1 > n_2$



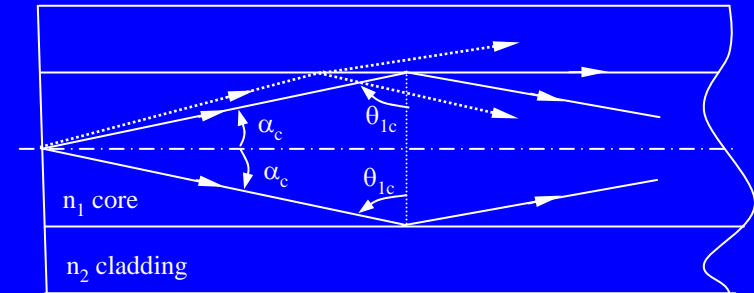
## ◆ Total Internal Reflection (全内反射)

Critical incident angle :  $\theta_{1c}$

(临界入射角)

Critical propagation angle :  $\alpha_c$

(临界传播角)



To save light inside an optical fiber, the propagation angle of rays should have

$$\alpha_z \leq \alpha_c .$$

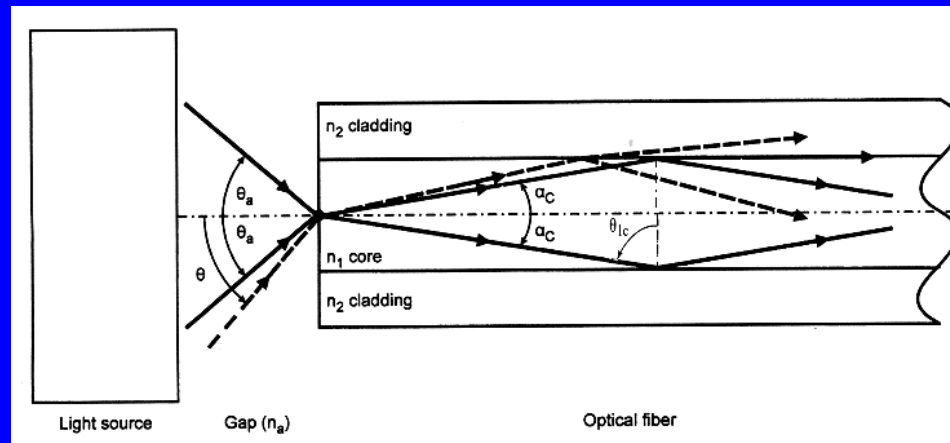
From Snell's Law, one has:

$$\sin \theta_{1c} = n_2/n_1 , \text{ and } \cos \alpha_c = n_2/n_1 .$$

Thus, one can derive:

$$\alpha_c = \sin^{-1} \sqrt{[1 - (n_2 / n_1)^2]}$$

## ◆ Launching the Light



Critical incident angle :  $\sin \theta_{lc} = \frac{n_2}{n_1}$

Critical propagation angle :  $\alpha_c = \sin^{-1} \sqrt{1 - \left( \frac{n_2}{n_1} \right)^2}$

-> Acceptance angle (接收角):  $\theta_a$  or  $2\theta_a$

$$\sin \theta_a = n_1 \sin \alpha_c$$

## ◆ Numerical Aperture (数值孔径)

$NA = \sin \theta_a$  : Describe the ability of an optical fiber to gather light from a source, and preserve this light inside the fiber.

$$= n_1 \sin(\alpha_c)$$

$$= n_1 \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2} = \sqrt{(n_1)^2 - (n_2)^2}$$

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$n = (n_1 + n_2) / 2$  : Average refractive index.

$\Delta = (n_1 - n_2) / n$  : Relative difference of the refractive indexes.

->  $NA = n\sqrt{2\Delta}$

:  $n_1$  and  $n_2$  are not important in themselves, but only in their average and relative difference.

## ◉ Example:

Type-1:

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$$n_1 = 1.48$$

$$n_2 = 1.46$$

$$\frac{\Delta n}{n} = 1\%$$

$$\Theta_{1C} = 80.57^\circ$$

$$\alpha_C = 9.43^\circ$$

$$\Theta_a = 14.033^\circ$$

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$$\text{NA} = 0.2425$$

Type-2:

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$$n_1 = 1.495$$

$$n_2 \approx 1.402$$

$$\frac{\Delta n}{n} \approx 6\%$$

$$\Theta_{1C} = 69.68^\circ$$

$$\alpha_C = 20.32^\circ$$

$$\Theta_a = 31.27^\circ$$

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$$\text{NA} = 0.5192$$



## ◆ Measurement of NA

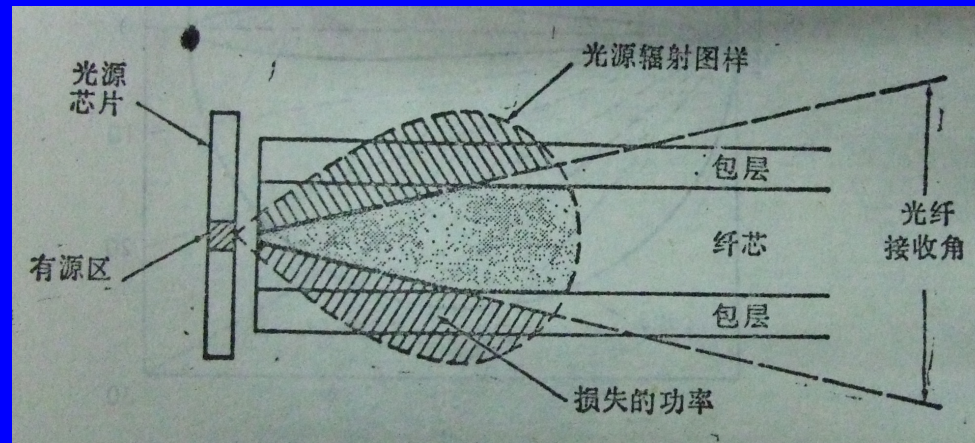
-> for a fiber launched by a LED source  
(Lambertian source: 朗伯光源)

$$P(\theta) = P_0 \cos \theta$$

$$\frac{P_{in}}{P_0} = \frac{\int_0^{2\pi} \int_0^{\theta_a} P_0 \cos \theta \sin \theta d\theta d\varphi}{\int_0^{2\pi} \int_0^{\frac{\pi}{2}} P_0 \cos \theta \sin \theta d\theta d\varphi} = \frac{\sin^2 \theta_a}{\sin^2 \frac{\pi}{2}} = (NA)^2$$

->

$$NA = \sqrt{P_{in} / P_0}$$



点光源发出球面波，Poynting矢量表达为

$$\mathbf{S}(r, \theta, \phi) = B(\theta, \phi) \frac{1}{r^2} \mathbf{r}$$

其中 $(r, \theta, \phi)$ 为球坐标，设点源位于坐标原点， $1/r^2$ 来自能量守恒要求， $\mathbf{r}$ 为点源到场点连线单位矢量。设球坐标系下曲面A方程为

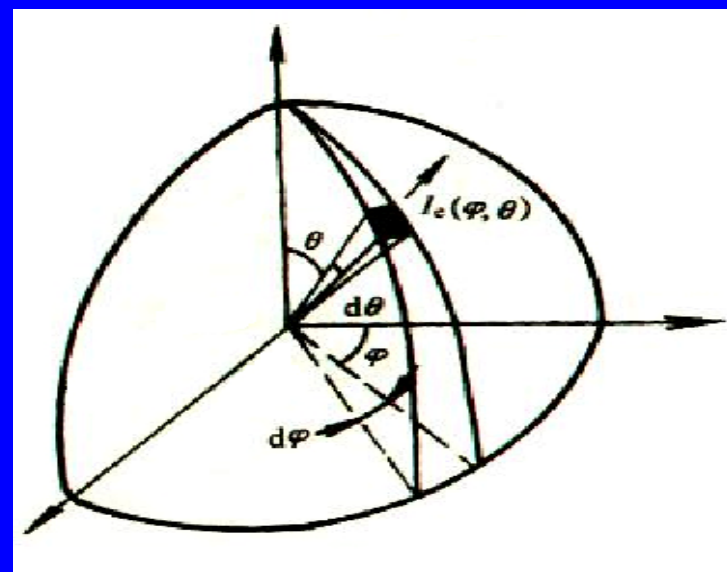
$$r = r(\theta, \phi), \theta \in [\theta_1, \theta_2], \phi \in [\phi_1, \phi_2]$$

则曲面A上流过的能流通量为

$$\Phi = \iint_A \mathbf{S}(r, \theta, \phi) \cdot \mathbf{n} dA = \iint_A B(\theta, \phi) \frac{1}{r^2} dA_n$$

$$dA_n = r^2 \sin \theta d\theta d\phi \quad (r \sin \theta d\phi \times r d\theta)$$

$$\Phi = \int_{\theta_1}^{\theta_2} \int_{\phi_1}^{\phi_2} B(\theta, \phi) \sin \theta d\theta d\phi$$



## ❏ Question :

Numerical aperture, NA, is the characteristic of an optical fiber to gather light from a source.

When the core diameter is larger, NA should be

- a. larger*
- b. smaller*
- c. the same*



# Nobel Prize in Physics

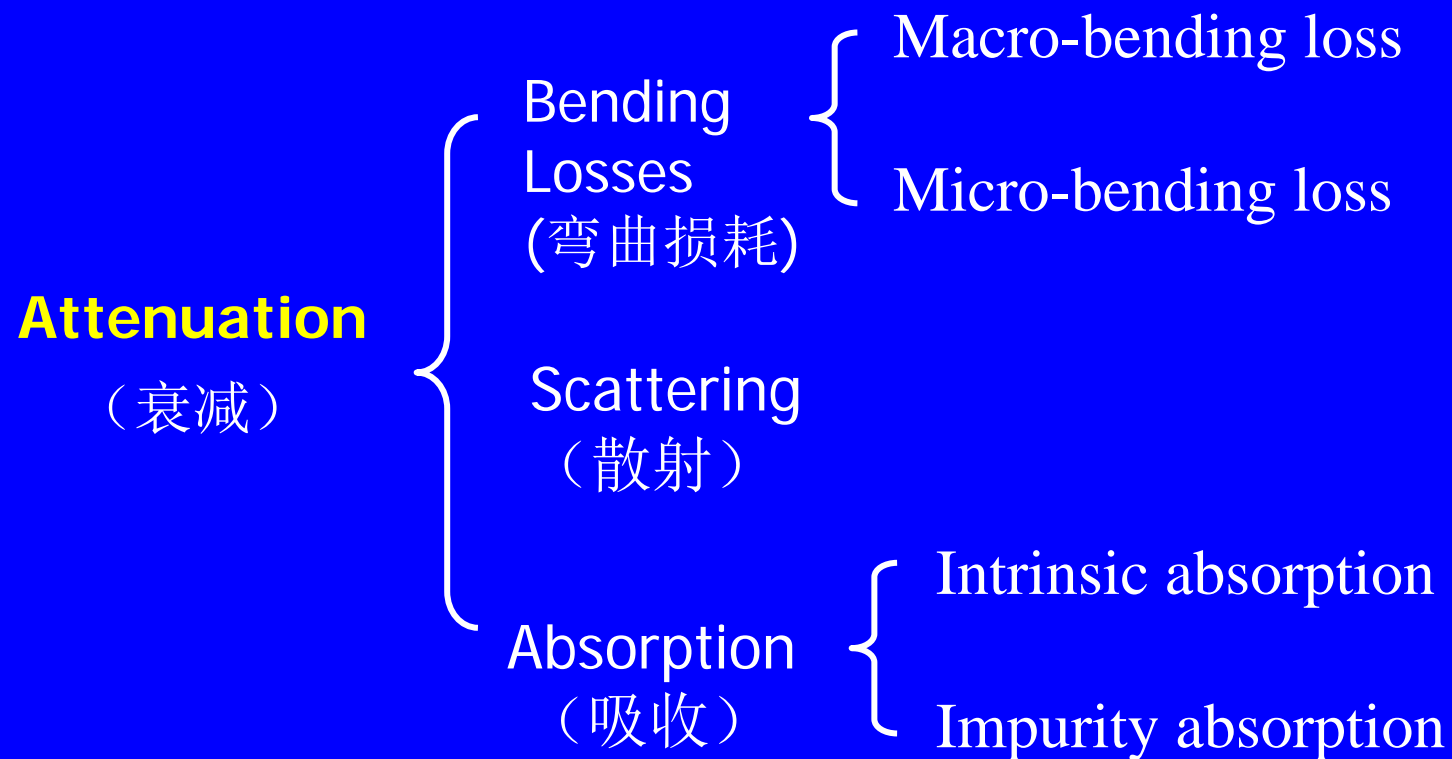
In 1965, **Charles K. Kao** with Hockham concluded that the fundamental limitation for glass light attenuation is below **20 dB/km** (*decibels per kilometer*, is a measure of the attenuation of a signal over a distance), which is a key threshold value for optical communications.

Attenuation !!



## 3.2 Attenuation

*: power loss for reasons other than failure to achieve total internal reflection.*



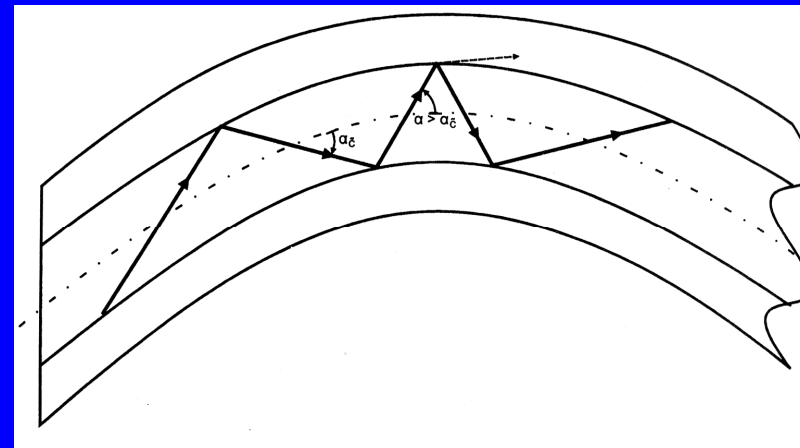
## ◆ Bending loss

- > The flexibility of optical fiber is an advantage, but it brings some problems too!
- > It will induce the failure of TIR.

### ● Macro-bending:

The curvature of the entire fiber changes the propagation angle to be more than critical angle.

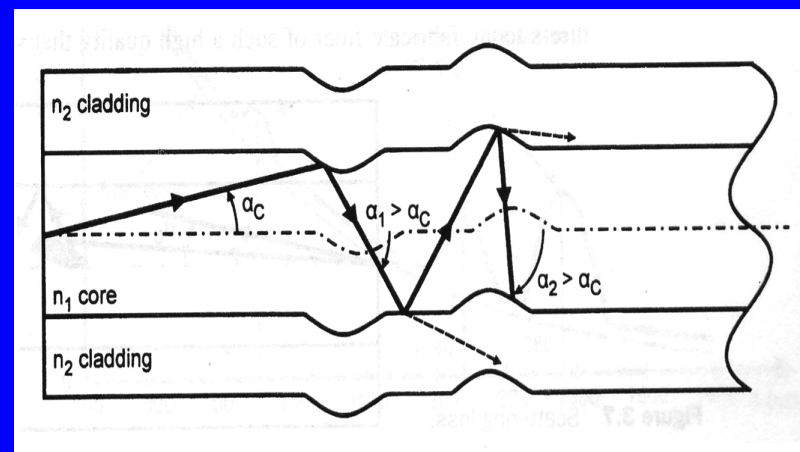
- > minimum bending radius:  
19mm or 13mm



### ● Micro-bending:

The microconvexity or microdent will change the propagation directions.

- > coating or external force

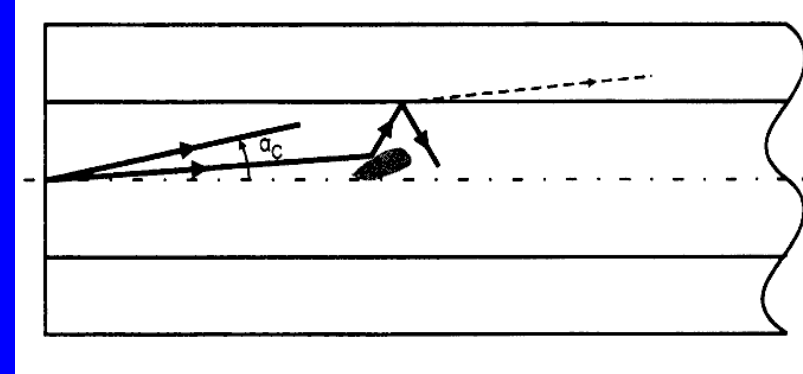


## ◆ Scattering loss

Change of propagation direction induced by small changes in the core's refractive index

-> induce the failure of TIR again

Rayleigh scattering :  $\propto \lambda^{-4}$



## ◆ (Material) Absorption

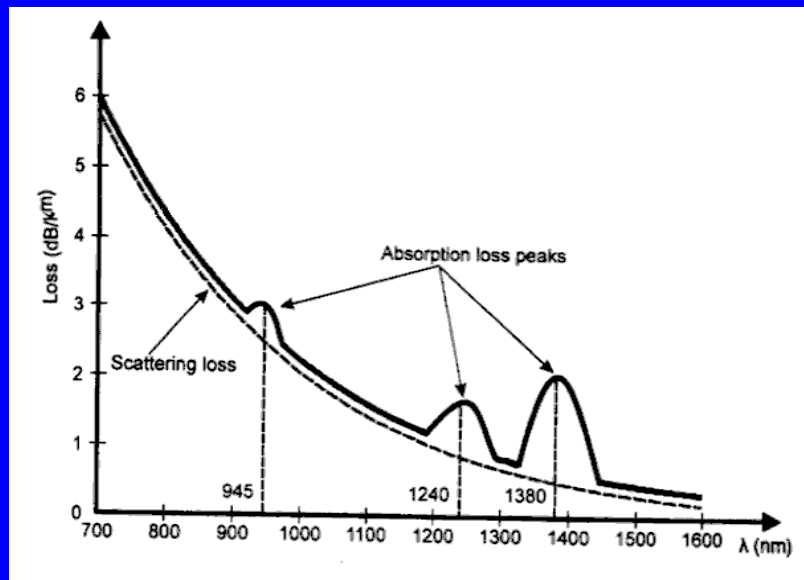
OH<sup>-</sup> molecules: 3 absorption peaks

Transparent windows:

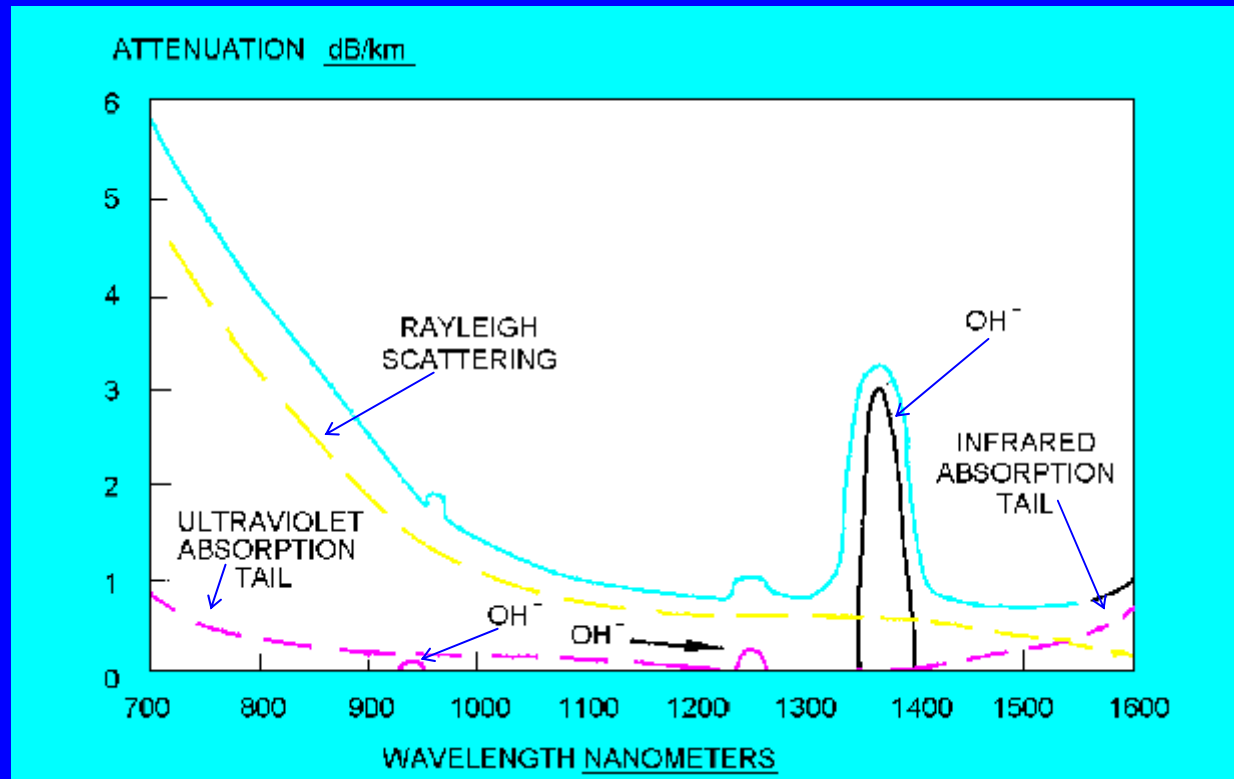
850 nm: 4 dB/km

1300nm: 0.5 dB/km

1550 nm: 0.3 dB/km



## ◆ Attenuation of Optical Fibers



-> : Dry fiber (无水光纤), or all-wave fiber (全波光纤), is optical fiber which hydroxide anion is eliminated.



## ◆ Calculation of Total Attenuation

- Loss in linear or decibels (分贝): (fiber, devices, ...)

$$Loss = P_{out} / P_{in}$$

$$Loss(dB) = -10 \log_{10} (P_{out} / P_{in}) \quad : \text{notice the minus sign!!}$$

- Attenuation (loss) per fiber-length : (fiber!)

(also called “Attenuation”, cable-loss factor, attenuation coefficient)

$$A(dB / km) = loss(dB) / fiberlength(km)$$

$$P_{out} = P_{in} \times 10^{-AL/10}$$

- **Maximum transmission distance**

$$L = (10/A) \log_{10} (P_{in} / P_{out})$$

-> the maximum transmission distance imposed by attenuation,

-> the minimum value of  $P_{out}$  is determined by the sensitivity of the receiver

- **Power unit *dBm*** : choose 1mW as the reference power

$$P_{out}(dBm) = +10 \log(P_{out} / 1mW)$$

   
(line 4, p. 56)

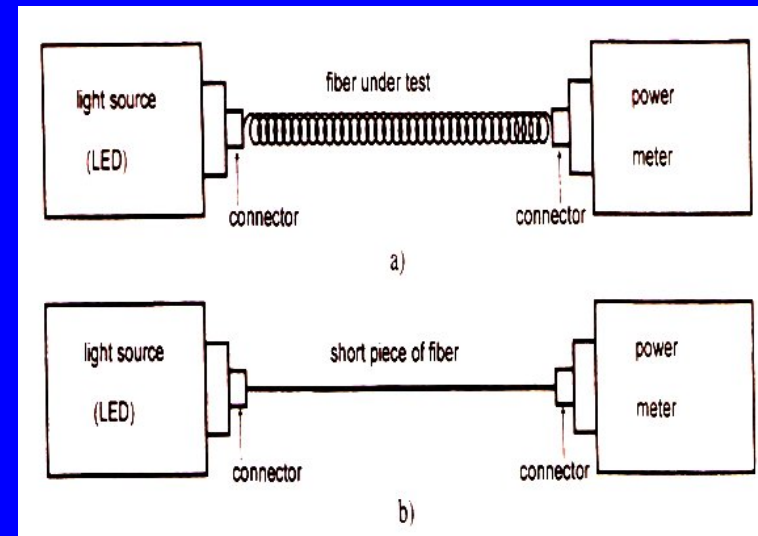
**P55, Paragraph 2 from the second section: “First, it is a key.....”**

## ◆ Measuring Attenuation

### **Cut Method** (截断法)

Measuring fiber attenuation with connection losses →

Measuring connection losses →



$$Loss(dB) = P_{out}(dBm) - P_{in}(dBm)$$

-> Measure the powers when the device under the test is “in and out”.

## *More about **Cut Method** (截断法) :*

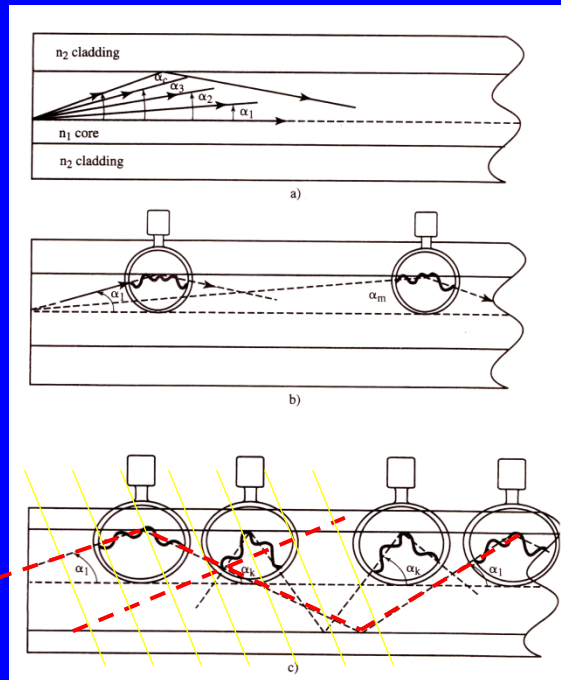
-> The precision of this method is mainly determined by two factors:

- (a) How accuracy you can reproduce connection losses
- (b) How negligible is the attenuation introduced by a short piece of fiber.

-> When measuring attenuation in **a multimode fiber**, special care should be taken to use a light beam filling the entire cross-sectional area of the core (called overfilled launching) to make sure that (all possible modes are excited).

## 3.3 Intermodal and Chromatic Dispersion

### ◆ Modes (模式)



- > Modes as different beams with different propagation distances.
- > Different beams experience different phase shifts (1. Different phase fronts. 2. Reflection phase shift.)
- > Optical fiber supports only those modes that complete the full zigzag at the same repeated phase (stable!). The other one is not stable.

Wave  $\alpha_1$  reproduces itself after the whole cycle of propagation, but wave  $\alpha_k$  does not.

## *More about **Modes** :*

- > These different beams with different propagation angles are called **modes**.
- > We distinguish modes by their propagating angles and we use the word **order** to designate the specific mode.
- > The smaller the mode's propagating angle, the lower the order of the mode.
- > The zero-order mode is also called **fundamental mode**.
- > The mode traveling at critical propagation angle is the highest order mode possible for this fiber.

## ◆ V-number (V参数, 归一化截止频率)

(Normalized cut-off frequency, characteristic waveguide parameter)

$$V = \frac{\pi d}{\lambda} NA$$

-> More light can be accommodated for larger core diameter d, larger NA and shorter wavelength

$$V = \frac{\pi d}{\lambda} \sqrt{(n_1)^2 - (n_2)^2}$$

->  $n_1, n_2$ : refractive indexes of the core and cladding

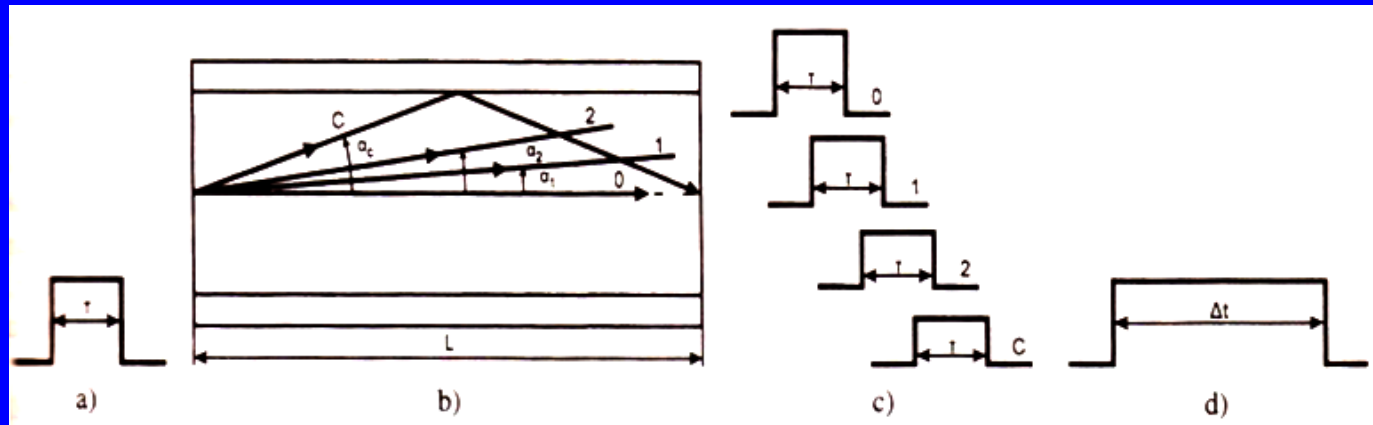
For a large V number ( >20 ), the number of modes can be estimated by

⊗ Step-index fiber:  $N = V^2 / 2$

⊗ Graded-index fiber:  $N = V^2 / 4$

## ◆ Intermodal dispersion (模间色散)

(different modes travels at different speeds)



a) Original pulse; b) Modes in an optical fiber; c) Pulses delivered by an individual mode; d) Resulting pulse

-> The zero-order (fundamental) mode needs time :  $t_0 = L/v$  ,  $v = c/n_1$  ;

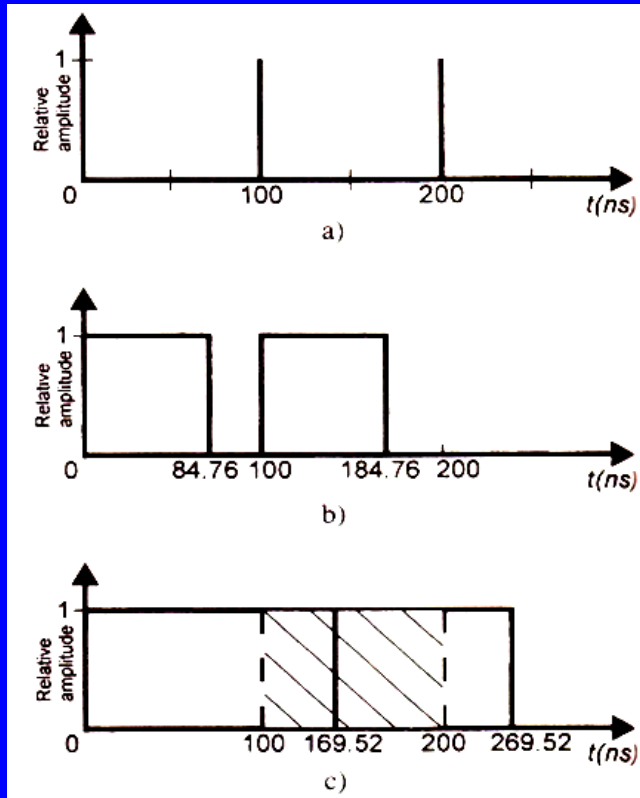
-> The highest-order (critical) mode needs time :  $t_c = L/v \cos \alpha_c$  (  $\cos \alpha_c = n_2/n_1$  )

⊗ Pulse spreading (脉冲展宽) stemming from intermodal dispersion:

$$\Delta t_{SI} = t_c - t_0 = \frac{L}{2cn_2} (NA)^2 \quad : \quad NA=0.275, \quad n_1 = 1.487 \Rightarrow \Delta t/L = 84.76 \text{ ns/km}$$



## ◆ Restriction on Bit Rate



a) Input pulses;

*Bit rate : 10Mbit/s,  
=> time cycle duration : 100 ns*

b) Pulses after 1 km transmission

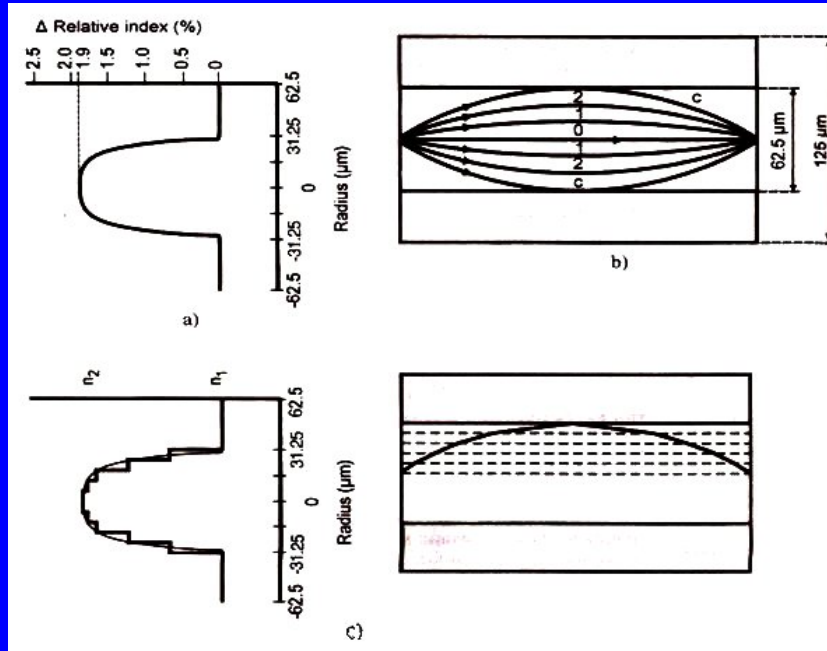
*Pulse spreading:  $\Delta t = 84.76$  ns*

c) Pulses after 2 km transmission

-> The maximum bit rate (for a transmission of 1 km)  
 $= 1/\Delta t = 11.8\text{Mbit/s}$ .

-> Larger  $L \Rightarrow$  larger  $\Delta t \Rightarrow$  smaller bit rate.

## ◆ First Solution: Graded-index Fiber (渐变光纤)



a) Refractive index profile

b) Mode propagation

c) Principle of action (fabrication)  
of graded index multimode fiber.

- > The refractive index of fiber core varies with the radius;
- > The beam traveling the farthest distance has the highest velocity and the beam traveling the shortest distance propagates at the slowest velocity.

- Calculating pulse spreading for graded-index fiber

$$\underline{\Delta t_{GI} = (LN_1\Delta^2)/(8c)}$$

\* where  $N_1$  is core group index of refraction  
 ->  $n_1$  is a variable, i.e.  $n_1(r)$

$$\Delta t_{GI} = \frac{[L(NA)^4]}{(32cN_1^3)}$$

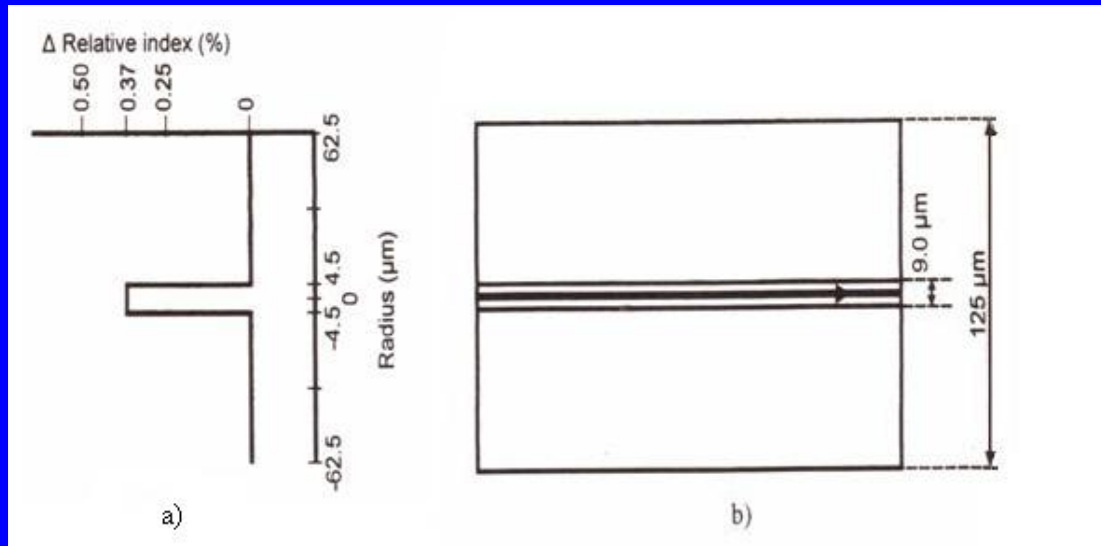
where approximation  $n_1 = N_1$  was used.

$$\Delta t_{GI} = \underbrace{\Delta t_{SI}}_{\Delta/8}$$

: A graded-index fiber has a modal dispersion  $\Delta/8$  times less than that of a step-index fiber.

\* Joseph C. Palais, Fiber Optic Communications, 4<sup>th</sup> ed., Englewood Cliffs, N.J.: Prentice Hall, 1998.

## ◆ A Better Solution : Single-mode Fiber (单模光纤)



Step-index Single Mode  
Optical fiber

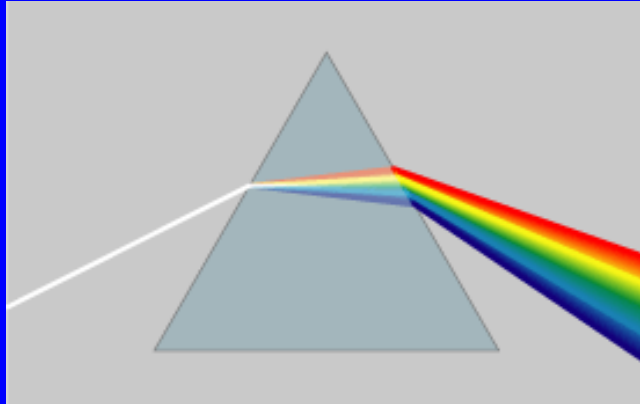
a) Refractive index profile

b) Mode propagation

-> Typically, the core diameter  $d$  is 8.3  $\mu\text{m}$  and the relative index  $\Delta$  is 0.37%.  
(for multimode fiber,  $d$  is around 62.5  $\mu\text{m}$ , and  $\Delta$  is 2%)

-> A real single-mode condition is :  $V \leq 2.405$

## ◆ Chromatic Dispersion (色度色散)



Why ?

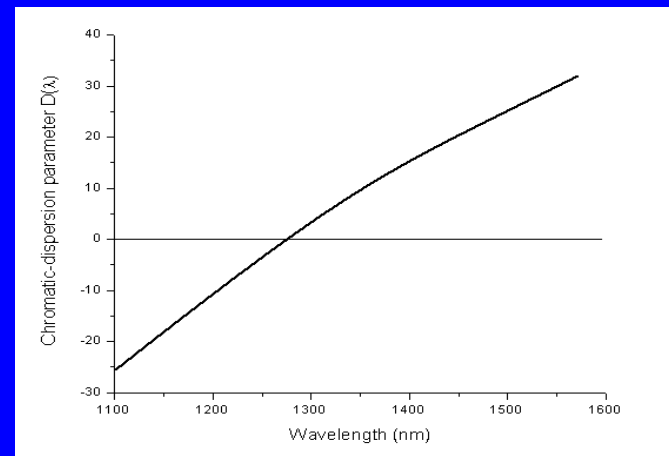
- > Refractive index depends on wavelength,  $n = n(\lambda)$
- > The velocity of light within a material is:  $v = c/n$
- > The light with different wavelength travels along the fiber at different velocities.
- > They will arrive at the receiver end at different times,  
even if all of these beams propagates along the same path.

- Calculating pulse spreading

$$\Delta t_{chrom} = D(\lambda) \cdot L \cdot \Delta \lambda$$

where  $D(\lambda)$  is the chromatic-dispersion parameter (色散参数) in ps/nm.km.

$$D(\lambda) = \frac{S_0}{4} \left[ \lambda - \frac{\lambda_0^4}{\lambda^3} \right]$$



->  $\lambda_0$  is the zero-dispersion wavelength : the wavelength at which  $D(\lambda)$  is zero.

->  $S_0$  is the zero-dispersion slope in ps/(nm<sup>2</sup>.km).

- Total pulse spreading caused by modal and chromatic dispersion

$$\Delta t_{total} = \sqrt{(\Delta t_{modal}^2 + \Delta t_{chrom}^2)}$$

## 3.4 Bit Rate (比特率) and Bandwidth (带宽)

- **Bit rate:** The number of bits that can be transmitted per second over a channel.

$$BR = \frac{1}{T} (Hz) \quad : T \text{ — time interval between adjacent signals.}$$

- **Bandwidth:** The frequency range within which a signal can be transmitted without significant deterioration.

$$\underline{BW = BR} \quad \text{or,} \quad BW = BR / 2$$

which depends on the line codes, such as the non-return to zero (NZR) *etc.*.

- **Dispersion and Bit Rate**

*for a practical standpoint, a coefficient 1/4 is general accepted in the industry.*

$$BR < 1 / (4\Delta t) \quad \text{i.e.} \quad T > 4\Delta t$$

-> for a step-index multimode fiber, one has

$$BR_{SI} = 1 / (4\Delta t_{SI}) = c / (4Ln_1\Delta) = cn_2 / (2LNA^2)$$

-> for a graded-index multimode fiber, one has

$$BR_{GI} = 2c / (N_1L\Delta^2)$$

-> If considering chromatic dispersion, one has

$$BR_{chrom} = 1 / (4D(\lambda)L\Delta\lambda)$$

-> If considering the total bit rate, one has

$$BR_{total} = 1 / 4\sqrt{(\Delta t_{modal}^2 + \Delta t_{chrom}^2)}$$



## ◆ Reading a data sheet

Data sheet maybe different from each other, but four parts must be included.

- “Optical Characteristics” section
- “Geometric Characteristics” section
- “Environmental Specifications” section
- “Mechanical Specifications” section

## ◆ Homeworks

### ◆ 3.7, 3.12, 3.21, 3.22, 3.25, 3.29, 3.30, 3.33, 3.47

- **3.7** The core refractive index is 1.4513 and the cladding index is 1.4468. What is (1) the critical propagation angle? (2) the acceptance angle? (3) the numerical aperture?
- **3.12** For a specific fiber,  $NA = 0.2375$  and  $n_1 = 1.4860$ . Find  $n_2$  (n cladding).
- **3.21** What does the term “transparent windows” mean? Specify three peak wavelengths for the transparent windows in modern optical fibers.
- **3.22** An optical fiber with attenuation of 0.25 dB/km is used for 20-km transmission. The light power launched into the fiber is 2mW. What is the output power?
- **3.25** Find the maximum transmission distance for a fiber link with an attenuation of 0.3 dB/km if the power launched in is 3mW and the receiver sensitivity is 100  $\mu$ W.

## ◆ Homeworks

- **3.29** What is the number of modes for a graded-index fiber if  $d$  is 50  $\mu\text{m}$ ,  $NA$  is 0.200, and the operating wavelength is 1300 nm?
- **3.30** How many modes can support a step-index optical fiber whose  $d=8.3 \mu\text{m}$ ,  $n_{\text{core}}=1.4513$ ,  $n_{\text{clad}}=1.4468$ , and  $\lambda=1550 \text{ nm}$ ?
- **3.33** Consider modal dispersion. For a step-index multimode fiber with  $NA=0.200$  and  $n_1=1.486$ : a) Evaluate pulse spreading per 1 km length; b) Calculate the maximum number of bits per second that can be transmitted over 1 km.
- **3.47** A graded-index fiber has  $n_1 = 1.486$  and  $NA = 0.200$ . What is the bit rate for a 1-km link?

## Reference:

1. Keiser, Gerd., Optical fiber communications, Publisher Boston, Mass. : McGraw-Hill, c2000.
2. G. P. Agrawal, Fiber-optic communication systems, Publisher New York : John Wiley, c2002
3. Haus, Hermann A., Waves and fields in optoelectronics, Publisher Englewood Cliffs, NJ : Prentice-Hall, c1984.

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